Derivatives Drill Sheet
with short solutions

Find the derivative for each function.
General tips: simplify using algebra first, apply derivative rules carefully.

1. $f(x) = \dfrac{Cx^2e^{-x} - \sqrt{x}}{x}$ (Hint: x is the variable here, so C must be a constant.)
   $f(x) = Cxe^{-x} - x^{-1/2}$, $f'(x) = e^{-x} - xe^{-x} + \frac{1}{2}x^{-3/2}$
2. $f(u) = \frac{\sin(u^2)}{1+\cos(u)}$ $f'(u) = \frac{(1+\cos(u))\cos(u^2)2u + \sin(u)\sin(u^2)}{(1+\cos(u))^2}$
3. $f(y) = \frac{1}{2y\sqrt{y}} + \frac{1}{4}r$ (Hint: What is the variable? What is the constant?)
   $f(y) = \frac{1}{2}y^{-3/2} + \frac{1}{4}r$, $f'(y) = -\frac{3}{4}y^{-5/2}$ (r is a constant here, so its derivative is zero)
4. $g(z) = \frac{4z}{6z^2+z^3}$ $g(z) = \frac{4}{6z+z^2}$, $g'(z) = \frac{-4}{(6z+z^2)^2} \cdot (6+2z)$
5. $f(x) = \ln(3x^2 + x^4)$ $f'(x) = \frac{6x+4x^3}{3x^2+x^4}$
6. $g(y) = e^{y^2\cos(2y)}$ $g'(y) = e^{y^2\cos(2y)}\left(2y\cos(2y) - 2y^2\sin(2y)\right)$
7. $F(u) = \sqrt{u}(1+\sqrt{u})$ $F(u) = u^{1/2} + u$, $F'(u) = \frac{1}{2}u^{-1/2} + 1$
8. $f(x) = 2^x + \left(\frac{1}{3}\right)^x$ $f(x) = 2^x + 3^{-x}$, $f'(x) = \ln(2)2^x - \ln(3)3^{-x}$
9. $f(r) = \pi + r^\pi + e^\pi + \pi^r$ $f'(r) = 0 + \pi r^{\pi-1} + 0 + \ln(\pi)\pi^r = \pi r^{\pi-1} + \ln(\pi)\pi^r$
10. $F(v) = \frac{e^v}{v+\sec(v)\tan(v)}$ $F'(v) = \frac{e^v(v+\sec(v)\tan(v)) - e^v\left(1+\sec(v)\tan^2(v)+\sec^3(v)\right)}{(v+\sec(v)\tan(v))^2}$
11. $f(x) = \frac{e^{-\tan(-x)}}{x+1}$ $f(x) = \frac{1}{e^{\tan(-x)}(x+1)}$, $f'(x) = \frac{-1}{(e^{\tan(-x)}(x+1))^2} \cdot \left(-\sec^2(-x)e^{\tan(-x)}(x+1) + e^{\tan(-x)}\right)$
12. $G(z) = \mathrm{Arctan}(\mathrm{e}^{2\mathrm{z}})$ $G'(z) = \frac{1}{1+e^{4z}} \cdot 2e^{2z}$
13. $f(x) = x\mathrm{Arcsin}(\mathrm{x})$ $f'(x) = \mathrm{Arcsin}(\mathrm{x}) + \frac{\mathrm{x}}{\sqrt{1-\mathrm{x}^2}}$
14. $f(z) = \mathrm{Arcsec}(\ln(\mathrm{z}))$ $f'(z) = \frac{1}{|ln(z)|\sqrt{(ln(z))^2-1}} \cdot \frac{1}{z}$
15. $f(x) = \frac{1}{(e^{-\pi}+1)^2}$ $f'(x) = 0$
16. $f(w) = \ln(\ln(w))$ $f'(w) = \frac{1}{\ln(w)}\frac{1}{w}$

Anti-Derivatives Drill Sheet
with short solutions

Find the following antiderivatives.
General tips: simplify using algebra first, apply antiderivative rules carefully.

1. $\int \frac{Cx^2e^{-x} - \sqrt{x}}{x}\, dx$
$= \int Cxe^{-x} - x^{-1/2}dx = \int Cxe^{-x}dx - 2x^{1/2}$. On the remaining integral use IBP with $u = x$, $du = dx$, $dv = e^{-x}dx$, $v = -e^{-x}$. This gives $uv - \int vdu - 2x^{1/2} = -xe^{-x} - \int -e^{-x}dx - 2x^{1/2}$. The final answer is $C(-xe^{-x} - e^{-x}) - 2x^{1/2} + D \overset{or}{=} -C\left(\frac{(x+1)}{e^x} + 2\sqrt{x}\right) + D$.

2. $\int \frac{\sin(u)}{1+\cos(u)}\, du$
Substitution with $w = 1 + \cos(u)$, $dw = -\sin(u)du$ gives $\int -\frac{1}{w}dw = -\ln|w| + C = -\ln|1 + \cos(u)| + C$.

3. $\int \frac{1}{2y\sqrt{y}} + \frac{1}{4}r\, dy$
$= \int \left(\frac{1}{2}y^{-3/2} + \frac{1}{4}r\right) dy = -\frac{1}{\sqrt{y}} + \frac{yr}{4} + C$

4. $\int \frac{[\ln(x)]^7}{x}\, dx$
Substitution with $u = \ln(x)$, $du = \frac{1}{x}dx$ gives the integral $\int u^7du = \frac{1}{8}u^8 + C = \frac{1}{8}[\ln(x)]^8 + C$.

5. $\int \frac{20z+6z^2+2}{5z^2+z^3+z}\, dz$
Substitution with $u = 5z^2 + z^3 + z$, $du = (10z + 3z^2 + 1)dz$ gives the integral $\int \frac{2du}{u} = 2\ln|u| + C = 2\ln|5z^2 + z^3 + z| + C$.

6. $\int \frac{4z+6}{3z+z^2}$ dz
The denominator is a *factorable polynomial*. This gives $\int \frac{4z+6}{z(z+3)}\, dz$. As long as the denominator is factorable and the degree of the polynomial in the numerator is *less* than the degree in the denominator, PFD can work. The PFD setup is $\frac{4z+6}{z(z+3)} = \frac{a}{z} + \frac{b}{z+3}$. Get a common denominator on the right-hand side and then cancel the common denominator of $z(z + 3)$ on both sides of the equation in order to obtain $4z + 6 = a(z + 3) + bz$. Set $z = -3$ to obtain $-12 + 6 = -3b$ and then $b = 2$. Set $z = 0$ to obtain $6 = 3a$ so that $a = 2$. Now going back to the integral $\int \frac{2}{z} + \frac{2}{z+3} dz = 2\ln|z| + 2\ln|z + 3| + C \overset{\text{or}}{=} 2\ln|z^2 + 3z| + C$.

7. $\int \ln(3x^2)dx$
$= \int \ln(3) + 2\ln(x)dx = \int \ln(3)dx + 2\int \ln(x)dx = \ln(3)x + 2\int \ln(x)dx$. To integrate the log function do IBP with $u = \ln(x)$, $du = \frac{1}{x}dx$, $dv = dx$, $v = x$. This gives $\ln(3)x + 2\left(uv - \int vdu\right) = \ln(3)x + 2\left(x\ln(x) - \int x\frac{1}{x}dx\right) = \ln(3)x + 2\left(x\ln(x) - \int 1dx\right) = \ln(3)x + 2\left(x\ln(x) - x\right) + C$.

8. $\int 3ye^{y^2/2}\, dy$
Substitution with $u = y^2/2$, $du = ydy$ gives $3\int e^{y^2/2}ydy = 3\int e^u du = 3e^u + C = 3e^{y^2/2} + C$

9. $\int \sqrt{u}\left(1 + \sqrt{u}\right) du$
$= \int \left(u^{1/2} + u\right) du = \frac{2}{3}u^{3/2} + \frac{1}{2}u^2 + C$

10. $\int 2^x + \left(\frac{1}{3}\right)^x dx$
$= \int \left(2^x + 3^{-x}\right) dx = \int 2^x dx + \int 3^{-x}dx$. On the first integral use the general property $\int a^x dx = \frac{1}{\ln(a)}a^x + C$. On the second integral start by using substitution with $u = -x$, $du = -dx$. Then overall we have $\frac{1}{\ln(2)}2^x - \int 3^u du = \frac{1}{\ln(2)}2^x - \frac{1}{\ln(3)}3^u + C = \frac{1}{\ln(2)}2^x - \frac{1}{\ln(3)}3^{-x} + C$

11. $\int (\pi + r^\pi + e^\pi + \pi^r)\, dr = \pi r + \frac{1}{(\pi+1)}r^{\pi+1} + e^\pi r + \frac{1}{\ln(\pi)}\pi^r + C$

12. $\int \quad (e^{ev} + \sec(v/8)\tan(v/8))dv = \frac{1}{e}e^{ev} + 8\sec(v/8) + C$

13. $\int \frac{e^{-\tan(-x)}}{\cos^2(-x)}\, dx$
$= \int \sec^2(-x)e^{-\tan(-x)}dx$. Substitution with $u = -\tan(-x)$, $du = \sec^2(-x)dx$ gives the integral $\int e^u du = e^u + C = e^{-\tan(-x)} + C$.

14. $\int \frac{e^{2z}}{1+e^{4z}}\, dz$
Substitution with $u = e^{2z}$, $du = 2e^{2z}dz$ gives the integral $\frac{1}{2}\int \frac{1}{1+u^2}du = \frac{1}{2}\mathrm{Arctan(u)} + \mathrm{C} = \frac{1}{2}\mathrm{Arctan(e^{2z})} + \mathrm{C}$

15. $\int \mathrm{Arcsin(x)dx}$
IBP with $u = \mathrm{Arcsin(x)}$, $du = \frac{1}{\sqrt{1-x^2}}dx$, $dv = dx$, $v = x$ gives $uv - \int vdu = x\mathrm{Arcsin(x)} - \int \frac{\mathrm{x}}{\sqrt{1-\mathrm{x}^2}}\mathrm{dx}$. On the integral part now use substitution with $w = 1 - x^2$,

$dw = -2xdx$. This gives $x\text{Arcsin}(x) + \frac{1}{2}\int \frac{1}{\sqrt{w}}dw = \text{Arcsin}(x) + \frac{1}{2} \cdot 2w^{1/2} + C =$ $x\text{Arcsin}(x) + \sqrt{1-x^2} + C$.

16. $\int \frac{2x\cos(x^2)}{\sqrt{\sin(x^2)+4}}\, dx$

    Substitution with $u = \sin(x^2) + 4$, $du = 2x\cos(x^2)dx$ gives $\int \frac{du}{u^{1/2}} = 2\sqrt{u} + C =$ $2\sqrt{\sin(x^2) + 4} + C$

17. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

    Substitution with $u = (e^x + e^{-x})$, $du = (e^x - e^{-x})dx$ gives $\int \frac{du}{u} = \ln|u| + C =$ $\ln|e^x + e^{-x}| + C$.

18. $\int \frac{1}{P(P-M)}\, dP$

    Note that P is the variable and M is a constant. Using PFD the integral becomes $\frac{1}{M}\int \left(\frac{1}{P-M} - \frac{1}{P}\right) dP = \frac{1}{M}\left(\ln|P-M| - \ln|P|\right) + C$. You can also write the answer as $\frac{1}{M}\ln\left|\frac{P-M}{P}\right| + C$.